

Spin-liquid model of the sharp resistivity drop in $La_{1.85}Ba_{0.125}CuO_4$.

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We use the phenomenological model proposed in our previous paper [Phys. Rev. Lett. **98**, 237001 (2007)] to analyse the magnetic field dependence of the onset temperature for two-dimensional fluctuating superconductivity $T^{**}(H)$. We demonstrate that the slope of $T^{**}(H)$ progressively goes down as H increases, such that the upper critical field progressively increases as T decreases. The quantitative agreement with the recent measurements of $T^{**}(H)$ in $La_{1.85}Ba_{0.125}CuO_4$ is achieved for the same parameter value as was derived in our previous publication from the analysis of the electron self energy.

Recent experiments on $La_{1-x}Ba_xCuO_4$ at $x = 1/8$ [1] revealed a complex hierarchy of energy scales in this material. It displays a charge ordering transition at $T_{co} = 54K$, a spin ordering transition at $T_{spin} = 42K$ with a subsequent one order of magnitude drop in the in-plane resistivity, the Berezinskii-Kosterlitz-Thouless (BKT) transition to a two-dimensional superconductivity at $T_{BKT} = 16K$, a crossover from 2D to 3D regime around $10K$, and a transition to a true 3D superconductivity at $4K$. This hierarchy is summarized and discussed in detail in [2].

It turns out that the temperature T^{**} where the resistivity crossover occurs is sensitive to the c -axis magnetic field which separates this phenomenon separately from the spin ordering. In this paper, we address the issue of this crossover. The measurements performed in a magnetic field [1] revealed that (i) T^{**} marks the onset of fluctuational diamagnetism, and (ii) T^{**} decreases with the field. These two effects and the fact that the resistivity sharply drops T^{**} are consistent with the idea that T^{**} marks the onset of a fluctuational pairing regime without (quasi-) long-range superconducting order. The details of the system behavior near T^{**} , however, depend on the underlying model. The authors of [2] considered a model of weakly coupled parallel superconducting stripes. Within this model, T^{**} is the temperature at which the inter-stripe coupling becomes strong, and a vortex liquid is formed.

We propose another explanation, based on the model with a flat Fermi surface in the antinodal regions near $(0, \pi)$ and $(\pi, 0)$ points in the Brillouin zone [3]. Fermions in these regions form two quasi-1D spin liquids coupled by Josephson-type interaction. In this model, the pairing amplitudes in the antinodal regions are developed at $T^* \gg T^{**}$ due to the attractive interactions in the spin-liquid state, however, phase fluctuations at $T \gg T^{**}$ are effectively one-dimensional, and are pinned by the defects. At T^{**} , the Josephson coupling becomes sufficiently strong to lock the relative phase of the two order parameters at π , and the system response becomes two-dimensional. This leads to depinning of the phase

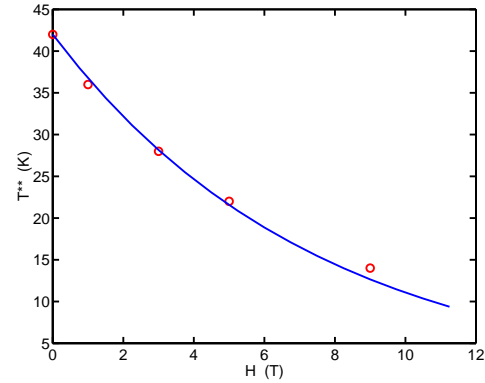


FIG. 1: The plot of $T^{**}(H)$. The points are the data from Ref. [1], the solid curve is the exponential fit by Eq. (1).

fluctuations resulting in the drop in the resistivity. Still, because of vortices in the 2D regime, the (quasi)-long-range superconducting order develops only at a smaller $T_c < T^{**}$.

Just like the model of parallel stripes[2], our model of “crossed stripes” near $(0, \pi)$ and $(\pi, 0)$ explains qualitatively the resistivity drop, the absence of fluctuational diamagnetism above T^{**} , and the sensitivity of T^{**} to a magnetic field. [5]. However, the measurements of $T^{**}(H)$ put an additional constraint on the theory – not only T^{**} decreases with the field, but $|dT^{**}/dH|$ also *decreases* as H goes up, i.e., at very low T , the critical field below which the system response is two-dimensional, becomes very large. The data for $H < 9T$ can be well fitted by the exponential dependence (see Fig. 1):

$$\frac{T^{**}(H)}{T^{**}(0)} = \exp(-H/H_0), \quad H_0 \approx 7.5T \quad (1)$$

For such $T^{**}(H)$, $|dT^{**}/dH|$ exponentially decreases as H increases. If this trend continued to higher H , the critical field $H_{c2}(T)$ defined as $T^{**}(H_{c2}) = T$ would become infinite at $T = 0$.

The H dependence of T^{**} for Josephson-coupled stripes running parallel to each other in the 2D plane,

i.e., for the same model as in Ref. [2] was considered by Carr and one of us [4]. It was found that the slope of dT^{**}/dH increases with decreasing T , and H_{c2} remains finite at $T = 0$, in qualitative disagreement with the data. We demonstrate below that our model of crossed stripes located near $(0, \pi)$ and $(\pi, 0)$ yields the behavior of $T^{**}(H)$ in a good agreement with the measurements. Thus we show that the slope of dT^{**}/dH decreases with increasing H for any value of the scaling dimension d of the superconducting order parameter. To achieve a quantitative agreement with the experimental fit (1) we have to set $d \approx 1/2$. We have to remind the reader that in [3]

the same value of d was postulated on the basis of analysis of the electron self energy. This gives an important check for self-consistency of the theory.

We associate $T^{**}(H)$ with the instability of a 2D pairing susceptibility in the random phase approximation (RPA). Fluctuations beyond RPA transform the instability into a crossover [3]. In zero field, the RPA expression for the susceptibility reads, in momentum space

$$\chi(k_x, k_y) = \chi_0(k_x) + J^2 \chi(k_x, k_y) \chi_0(k_x) \chi_0(k_y) \quad (2)$$

where $\chi_0(k)$ is the 1D static pairing susceptibility [6]:

$$\chi_0(k) = \frac{2}{\Delta^2} \left[\sin \pi d \Gamma^2(1-d) \left(\frac{2\pi T}{\Delta} \right)^{-2+2d} \left| \frac{\Gamma(d/2 + ivq/4\pi T)}{\Gamma(1-d/2 + ivq/4\pi T)} \right|^2 - \frac{\pi}{1-d} \right] \quad (3)$$

Here $\Gamma(\dots)$ are Γ -functions, $d < 1$ is the scaling dimension of the superconducting order parameter, v is the velocity of the phase mode, and Δ is the ultraviolet cut-off. The last term in χ_0 can be neglected as we will only consider $T \ll \Delta$, when the first term in (3) dominates. Parameters v and d are free parameters of our theory and should be extracted from the experiments in the T region where the superconducting phase fluctuations are essentially one-dimensional (that is, at T below the spin gap, but larger than T^{**}). In [3] we found that the best agreement with the photoemission experiments is obtained when $d \approx 1/2$. As we will see, this value is also favored by the observed $T^{**}(H)$ dependence.

Taking a Fourier transform over k_x , but leaving k_y intact, we obtain from (2):

$$\begin{aligned} \chi_{k_y}(x - x_1) = & \quad (4) \\ \chi_0(x - x_1) + J^2 \int dx' \chi_0(k_y) \chi_0(x - x') \chi_{k_y}(x' - x_1) \end{aligned}$$

In a magnetic field, $k_y \rightarrow k_y + Hx'$ (we set $2e/c = 1$). Setting $k_y = 0$ and $x_1 = 0$, we obtain integral equation for $\chi(x) = \chi_{k_y=0}(x)$ in the form

$$\chi(x) = \chi_0(x) + J^2 \int dx' \chi_0(x - x') \chi(x') \chi_0(Hx') \quad (5)$$

where $\chi_0(Hx')$ is given by (3) for $k = Hx'$, and $\chi_0(x)$ is the Fourier transform of $\chi_0(k)$. The temperature $T^{**}(H)$ is the one at which $\chi(x)$ diverges.

Weak fields. Consider first the case when the magnetic field is weak, i.e., $T^{**}(H) = T^{**}(0)(1 - \delta T)$, and $\delta T \ll 1$. A simple analysis shows that the parametrical condition for a weak field is $v^2 H/T \ll 1$. Expanding

$\chi_0(Hx')$ in H , we obtain from (3)

$$\chi_0(Hx') = B_d \left(\frac{2\pi T}{\Delta} \right)^{2d-2} \left[1 - A_d \left(\frac{vHx'}{\pi T} \right)^2 \right] \quad (6)$$

where

$$\begin{aligned} A_d &= \frac{1}{16} \left[\psi^{(1)}(d/2) - \psi^{(1)}(1-d/2) \right], \\ B_d &= \frac{2}{\Delta^2} \sin \pi d \Gamma^2(1-d) \frac{\Gamma^2(d/2)}{\Gamma^2(1-d/2)} \end{aligned} \quad (7)$$

and $\psi^{(1)}(x)$ is the derivative of the diGamma function.

Substituting (6) into (5), we obtain an integral equation for $\chi(x)$ in the form

$$\begin{aligned} \chi(x) = \chi_0(x) + J^2 \int dx' \chi_0(x - x') \chi(x') \chi_0(0) \\ - J^2 \chi_0(0) A_d \frac{v^2 H^2}{(\pi T)^2} \int dx' \chi_0(x - x') \chi(x') (x')^2 \end{aligned} \quad (8)$$

where $\chi_0(0) = \chi_0(k=0)$. Taking Fourier transform back to momentum space ($x \rightarrow k_x = k$), and integrating by parts, we re-write the integral equation for χ as

$$\begin{aligned} \chi(k) [1 - J^2 \chi_0(k) \chi_0(0)] - J^2 \chi_0(k) \chi_0(0) \frac{A_d v^2 H^2}{(\pi T)^2} \chi''(k) \\ = \chi_0(k) \end{aligned} \quad (9)$$

This can be re-expressed as

$$\left(\epsilon + c_1 k^2 - c_2 \frac{\partial^2}{\partial k^2} \right) \chi(k) = \chi_0(k) \quad (10)$$

where $\epsilon = 1 - (T^{**}(0)/T)^{4-4d}$, $c_1 = A_d v^2 / (\pi T)^2$, $c_2 = A_d v^2 H^2 / (\pi T)^2$, and we defined $T^{**}(0) =$

$(\Delta/2\pi) (B_d J)^{1/(2-2d)}$. This agrees with the zero-field transition temperature in [3]. Expanding now in the eigenvalues of the differential equation as

$$\chi(k) = \sum_n a_n \chi_n(k), \quad \chi_0(k) = \sum_n a_n^{(0)} \chi_n(k) \quad (11)$$

where $\chi_n(k)$ are the solutions of

$$\left(c_1 k^2 - c_2 \frac{\partial^2}{\partial k^2} \right) \chi_n(k) = \epsilon_n \chi_n(k), \quad (12)$$

we obtain

$$a_n = \frac{a_n^{(0)}}{\epsilon + \epsilon_n} \quad (13)$$

The eigenvalues of Eq. (12) can be easily obtained as (12) can be re-expressed as a harmonic oscillator

$$-\frac{1}{2M} \frac{\partial^2 \chi_n(k)}{\partial k^2} + \frac{M\omega^2 k^2}{2} \chi_n(k) = \epsilon_n \chi_n(k) \quad (14)$$

where $\omega^2 = 4c_1 c_2$ and $M^{-1} = 2A_d(v/\pi T)^2$. The eigenfunctions of (14) are $\epsilon_n = \omega(n + 1/2)$, the lowest one is $\epsilon_0 = \omega/2 = Av^2 H/(\pi T)^2$. From (13), the instability in the field occurs when $\epsilon + \epsilon_0 = 0$, i.e., when $T = T^{**}(H) = T^{**}(0)(1 - \delta T)$, where

$$\delta T \approx \frac{1}{4(1-d)} \frac{A_d v^2 H}{(\pi T^{**}(0))^2} \quad (15)$$

We see that at small fields, $T^{**}(H)$ decreases linearly with H . The linear dependence at small fields is also present in the model of parallel stripes [4]. If we formally extrapolate the small-field result to $T = 0$, we obtain the upper critical field

$$H_{c2}^{extr}(T=0) = \left(\frac{\Delta}{v} \right)^2 (JB_d)^{1/(1-d)} \frac{1-d}{A_d} \quad (16)$$

The actual $H_{c2}(T=0)$ is somewhat smaller in the model of parallel stripes [4], but, as we will see, is much larger than (16) in our model of crossed stripes.

Strong fields. Consider now the opposite limit of vanishing T , when $v^2 H/T \gg 1$, i.e., the expansion in the field is no longer possible. In this limit, we have from (3)

$$\chi_0(Hx') = \frac{\bar{B}_d}{|Hx'|^{2-2d}} \quad (17)$$

where

$$\begin{aligned} \bar{B}_d &= (8/\Delta)^2 \sin(\pi d) \Gamma^2(1-d) (v^2/4\Delta^2)^d \\ &= B_d (2\Delta/v)^{2-2d} (\Gamma^2(1-d/2)/\Gamma^2(d/2)). \end{aligned} \quad (18)$$

Instead of Eq. (9), we now have

$$\chi(k) = \chi_0(k) \left[1 + J^2 \frac{\bar{B}_{2d-1}}{H^{2-2d}} \int dq \chi(q) \int dx' \frac{e^{i(k-q)x'}}{|x'|^{2-2d}} \right] \quad (19)$$

Using

$$\int dx' \frac{e^{i(k-q)x'}}{|x'|^{2-2d}} = \frac{\Gamma(2d-1) \sin \pi d}{|k-q|^{2d-1}} \quad (20)$$

and introducing

$$\chi(k) = \frac{\bar{B}_d}{|k|^{2-2d}} \tilde{\chi}(k) \quad (21)$$

and $d = (1 + \epsilon)/2$, we obtain from (19)

$$\tilde{\chi}(k) = 1 + \frac{J^2 \bar{B}_d^2 \cos(\pi\epsilon/2) \Gamma(\epsilon)}{H^{1-\epsilon}} \int dq \frac{\tilde{\chi}(q)}{|q|^{1-\epsilon} |k-q|^\epsilon} \quad (22)$$

It is convenient to re-express this equation in the operator form, as $\hat{L}\tilde{\chi}(k) = 1$, and expand in the eigenfunctions of the operator \hat{L} , which we label as $\tilde{\chi}_m(k)$. We get

$$\tilde{\chi}(k) = \sum_m \frac{a_m^{(0)}}{1 - \lambda_m} \tilde{\chi}_m(k) \quad (23)$$

where $a_m^{(0)}$ are constants. The eigenvalues λ_m are the solutions of

$$\hat{L}\tilde{\chi}_m(k) = (1 - \lambda_m) \tilde{\chi}_m(k) \quad (24)$$

where

$$\hat{L}\tilde{\chi}_m(k) = \tilde{\chi}_m(k) - \frac{J^2 \bar{B}_d^2 \cos \pi\epsilon/2 \Gamma(\epsilon)}{H^{1-\epsilon}} \int dq \frac{\tilde{\chi}_m(q)}{|q|^{1-\epsilon} |k-q|^\epsilon} \quad (25)$$

Eq. (25) was studied in the context of non-BCS superconductivity (with frequency instead of momentum) [7]. A similar equation has been studied in the content of superconductivity in graphene [8]. For $\epsilon > 0$, the normalized solution of (25) with the largest eigenvalue is

$$\tilde{\chi}_m(k) = \frac{1}{|k|^\epsilon} \quad (26)$$

and the eigenvalue is

$$\lambda_0 = \frac{J^2 \bar{B}_d^2}{H^{1-\epsilon}} \Psi_\epsilon, \quad \Psi_\epsilon = \frac{\pi^2}{2} \frac{1}{\Gamma^2(1-\epsilon/2) (\sin \pi\epsilon/4)^2} \quad (27)$$

The critical field $H_{c2}(T=0)$ is determined from $\lambda_0 = 1$ and is given by

$$H_{c2}(T=0) = [J^2 \bar{B}_d^2 \Psi_\epsilon]^{1/(1-\epsilon)} \quad (28)$$

In explicit form, we have

$$\begin{aligned}
H_{c2}(T=0) &= (J\bar{B}_d)^{1/(1-d)} \left(\frac{2\Delta}{v}\right)^2 \left(\frac{8}{(2d-1)^2}\right)^{1/2(1-d)} \left[\frac{\Gamma(1-d/2)}{\Gamma(d/2)}\right]^{2/(1-d)} \\
&= H_{c2}^{extr}(T=0) \left[\left(\frac{4A_d}{1-d}\right) \left(\frac{8}{(2d-1)^2}\right)^{1/2(1-d)} \left[\frac{\Gamma(1-d/2)}{\Gamma(d/2)}\right]^{2/(1-d)}\right]
\end{aligned} \tag{29}$$

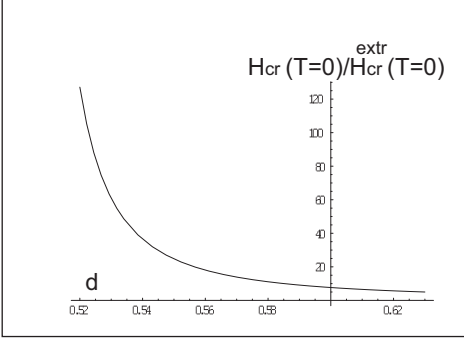


FIG. 2: The ratio $H_{c2}(T=0)/H_{c2}^{extr}(T=0)$ as a function of scaling dimension d , from Eq.(29). The ratio diverges logarithmically at $d \rightarrow 0.5$.

One can easily make sure that the actual $H_{c2}(T=0)$ is much larger than $H_{c2}^{extr}(T=0)$ for all $d \leq 1/2$ for which our computational scheme is applicable. Furthermore, as d approaches $1/2$, $H_{c2}(T=0)$ tends to infinity because $\Psi(\epsilon)$ diverges at vanishing $\epsilon = 2d - 1$ as $\Psi_\epsilon \approx 8/\epsilon^2$. The plot of the ratio $H_{c2}(T=0)/H_{c2}^{extr}(T=0)$ is presented in Fig. 2.

For $d \leq 1/2$, the analysis has to be modified to account for the divergence at $q = 0$ in the r.h.s. of (25). The expected result is that H_{c2} becomes infinite at zero temperature. The divergence is power-law for $\epsilon < 0$, and logarithmical at $\epsilon = 0$. In the latter case,

$$\chi_0(Hx') = \frac{\bar{B}_{\epsilon=0}}{|Hx'|} \tag{30}$$

and the RPA equation for $\chi(x)$ in the real space becomes

$$\chi(x) = -2 \log T|x| - \frac{2(J\bar{B}_{\epsilon=0})^2}{H} \int \frac{dx'}{|x'|} \chi(x') \log(T|x-x'|) \tag{31}$$

With the logarithmic accuracy, we can approximate

$$\log(|x-x'|) \approx \theta(x-x') \log|x| + \theta(x'-x) \log|x'| \tag{32}$$

Substituting into (31), we re-write it as a differential equation

$$\partial_\zeta^2 \chi + \frac{4(J\bar{B}_{\epsilon=0})^2}{H} \chi = -2\partial_\zeta^2 \log(T|e^\zeta - 1|) \tag{33}$$

where $\zeta = \log|x|$. The analysis of this equation shows that the susceptibility diverges at $H = H_{c2}(T) \propto |\log T|$. This is equivalent to $T^{**}(H) \propto \exp(-H/H_0)$, in agreement with Eq. (1). We see therefore that the high field dependence is well captured by our model with $d \approx 1/2$ – the same as we used in the previous work [3] to fit the normal state self-energy.

To summarize, we analyzed the behavior of $T^{**}(H)$ (or, equivalently $H_{c2}(T)$) in the model of two one-dimensional spin liquids near $(0, \pi)$ and $(\pi, 0)$ coupled by Josephson-type interaction. For weak fields we found that T^{**} decreases linearly with H . Extrapolating this dependence down to zero temperature yields the extrapolated field $H_{c2}^{extr}(T=0)$. Considering the strong fields we found that the actual $H_{c2}(T=0)$ is always larger than the extrapolated value. The ratio $H_{c2}(T=0)/H_{c2}^{extr}(T=0)$, characterizing the convexity of the $H_{c2}(T)$ -curve, increases when d decreases and becomes infinite at $d \leq 1/2$. This convex behavior is consistent with the data, and has to be contrasted with the *concave* behavior for the model of parallel stripes. As a further evidence in support of our model, we found that the experimental $H_{c2}(T)$ are well described by the theoretical formula with the scaling dimension of the 1D superconducting order parameter $d \approx 1/2$. The same d provides the best fit to the photoemission data, as we argued earlier [3]. We think that all these give our model a considerable advantage in treating $La_{1.85}Ba_{0.125}CuO_4$.

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